Assignment #3

Date Due: December 7, 2021

Total: 100 marks

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1. (10 marks) Write a grammar that generates the language

$$\{w \in \{a,b\}^* \mid |w|_a = |w|_b + 1\}.$$

2. (10 marks) Prove that the following grammar is ambiguous $S \to B{*}A|a\%A\%a,\ A \to A{*}A|a,$ $B \to b$.

3. (10 marks) Write an equivalent grammar for the following DFA

```
(START) |- 0        4 a 3
0 a 1               4 b 5
0 b 2               5 a 2
1 a 0               5 b 4
1 b 6               6 b 6
2 b 4               6 a 7
2 a 3               7 a 7
3 a 3               7 b 7
3 b 5               1 -| (FINAL)
                    4 -| (FINAL)
                    6 -| (FINAL)
```

4. (10 marks) Construct an equivalent DFA for the following grammar

```
S->aA               A->bB
S->bB               B->b
S->bS               B->bC
S->aC               C->bA
A->b                C->b
A->c                C->a
A->a                C->aB
                    C->aC
```

5. (20 marks maximum) Prove that the following languages are context free:
   (a) (10 marks) $\{a^{n+2}b^{m+3}c^{n+1} \mid m, n \geq 0\}$
   (b) (10 marks) $\{a^{n+2}b^{n+1}c^m d^{m+1} \mid m, n \geq 0\}$
   (c) (10 marks) $\{uc^n v \mid |v|_a + 2|v|_b = 2|u|_a + |u|_b, n \geq 1\}$
6. (10 marks) Given the following grammar:

```
S -> aS | bS |aAaA |BbAb
A -> aB | bC | a
B -> aA | bC |AB
C -> aA | bB | a  | b
```

   (a) Construct the PDA that accepts the same language by empty stack.
   (b) Construct an equivalent PDA that accepts the same language by final states.
7. (20 marks maximum) Prove that the following languages are not context free:
   (a) (10 marks) $\{a^{p-2} \mid p \text{ is prime}\}$
   (b) (10 marks) $\{a^{2n}b^{3n}c^{4n} \mid n \geq 4\}$
   (c) (10 marks) $\{a^{n^4} \mid n \geq 2\}$
8. (10 marks) A language is said to be a *palindrome* language if $L = L^R$. Find an algorithm for determining if for a given a DFA A, $L(A)$ is a palindrome language.

   Example of palindrome languages: $\{abb, bba\}$, $\{ab^n \mid n \geq 0\} \cup \{b^n a \mid n \geq 0\}$, $\{ab^n a \mid n \geq 0\}$. Example of languages that are ***not*** palindrome languages: $\{ab^n \mid n \geq 0\}$, $\{ab^n a \mid n \geq 0\} \cup \{(ba)^n \mid n \geq 0\}$, $\{abba, abb, bba, aab, ba\}$.

   Note that a palindrome language may contain no palindromes, but a language containing just palindromes is a palindrome language.
9. (10 marks) Let L be a palindrome language. Is L a regular language? Give an example of a palindrome language that is not regular (a proof is required).