Assignment #2

Date Due: November 15, 2021

Total: 150 marks

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1. (20 marks) We denote by

   $L_1 = \{$the set of all strings over the alphabet $\{0, 1\}$ that begin with 1010$\}$

   $L_2 = \{$the set of all strings over the alphabet $\{0, 1\}$ that end with 0101$\}$

   $L_3 = \{$the set of all strings over the alphabet $\{0, 1\}$ that begin with 00110$\}$ Give a regular expression for the following language over the alphabet $\{0, 1\}$:

   (a) $L_4 = L_1 \cap L_2$, and

   (b) $L_5 = L_3 \cap L_2$

   Transform the regular expressions into an equivalent ε-NFA, and afterwards in a DFA, using the algorithms learned in class (or the ones in your textbook).

   Minimize the resulting DFAs.

2. (40 marks) Give a regular expression for each of the following languages over the alphabet $\{0, 1\}$:

   (a) the set of all strings consisting of alternating groups of 11 and 101 (11 and 101 *alternates* at least once);

   (b) the set of all strings whose forth symbol from the right end is an 1;

   (c) the set of strings that either begin, or end (or both) with 0101;

   (d) the set of strings such that the number of 0's is divisible by six, and the number of 1's is not divisible by seven.

3. (20 marks) Write regular expressions for the following languages over the alphabet $\Sigma = \{0, 2, 3, 5, 7\}$:

   (a) the set of all strings beginning with a 7, 3 or 5, that, when the string is interpreted as an integer in base 9, is a multiple of 5 plus 2. For example:

      - strings 7, 30, 35, 52, 502, 5002, 5057, 50057, 705, and 77777 are in the language;
      - the strings 20, 00,022, 0020, 37 , 23, 33, 223, 2325, 2375, 3, 5, 33333,, 22222, 505, 22, 72, and 035 are not.

(b) The set of all strings that ends with an **7, 3, or 5** and when the string is interpreted ***in reverse*** as an integer **in base 9, is a multiple of 5 plus 2**.

Examples of strings in the language are 7, 03, 53, 25, 205, 2005, 7505, 75005, 507, and 77777 Examples of strings that are not in the language are: 02, 00,220, 0200, 73 , 32, 33, 322, 5232, 5732, 3, 5, 33333,, 22222, 505, 22, 27, and 530.

4. (10 marks each) Describe in English, as simple as possible, the languages generated by the following regular expressions:

(a) $a^*a(a+b^*)^*$

(b) $(b+a^*)^*b(b^*)^*$

(c) $(bb+\epsilon)(aabb)^*(aa+\varepsilon)$

(d) $b^*ab^*(b^*ab^*ab^*ab^*ab^*)^*$.

5. (25 marks) Prove that the following languages are regular languages:

(a) $\{a^nb^ma^k \mid n \geq 2, m \geq 2, k \geq 1\}$

(b) $\{a^n \mid n \neq 2 \text{ and } n \not\equiv 3(\mod 6)\}$

(c) $\{a^nb \mid n \geq 3\} \cup \{ab^m \mid m \geq 1\}$

6. (25 marks) Prove that the following languages are not regular:

(a) $\{c^mb^na^n \mid n > 1, m \geq 0\}$

(b) $\{ca^nb^{n+k+1}a^k \mid n > 0, n > k > 1\}$

(c) $L = \{c^ma^nb^l \mid n \neq l, m > 2\}$

7. (10 marks) Is the following language regular or not?

$$\{a^nb^la^m \mid |n-m| \leq 1, l > 2\}.$$