Assignment #1

Date Due: October 9, 2026

Total: 3 marks

We have the following languages:
$L_1 = \{$the set of all strings over the alphabet $\{0,1,2\}$ that begin with 1010$\}$,
$L_2 = \{$the set of all strings over the alphabet $\{0,1,2\}$ that end with 01011$\}$,
$L_3 = \{$the set of all strings over the alphabet $\{0,1,2\}$ having 010 a a subword$\}$,
$L_4 = \{$the set of all strings over the alphabet $\{0,1,2\}$ having an odd number of 0's$\}$,
$L_5 = \{$the set of all strings over the alphabet $\{0,1,2\}$ having an even number of 1's$\}$,
$L_6 = \{$the set of all strings over the alphabet $\{0,1,2\}$ having the fifth symbol from the right end a 1$\}$,
$L_7 = \{$the set of all strings over the alphabet $\{0,1,2\}$ beginning with 22021$\}$,
$L_8 = \{$the set of all strings over the alphabet $\{0,1,2\}$ ending in 22021$\}$,
$L_9 = \{$the set of all strings over the alphabet $\{0,1,2\}$ having the number of 1's multiple of 6$\}$,
$L_10 = \{$the set of all strings over the alphabet $\{0,1,2\}$ having the number of 1's multiple of 5$\}$,
$L_{11} = \{$the set of all strings over the alphabet $\{a,b\}$ having the number of a's multiple of 7$\}$,
$L_{12} = \{$the set of all strings over the alphabet $\{a,b\}$ having the number of b's multiple of 5$\}$,
$L_{13} = \{$the set of all strings consisting of alternating groups of 12 and 02
(12 and 02 *alternates* at least once)$\}$,
and the following homomorphisms
$h : \{a,b\} \longrightarrow \{0,1,2\}^*$, $g : \{0,1,2\} \longrightarrow \{a,b\}^*$, $h(a) = 01$, $h(b) = 21$, $g(0) = a$, $g(1) = ba$.

1. (5 ok) Compute the languages
   (a) $L_{20} = L_1 \cap L_2$.
   (b) $L_{21} = 01011\Sigma^* \cap \Sigma^*1010$
   (c) $L_{22} = L_{13}$
   (d) $L_{23} = L_6$
   (e) $L_{24} = L_7 \cap L_8$
   (f) $L_{25} = L_{11} \setminus L_{12}$
   (g) $L_{26} = h^{-1}(L_4)$
   (h) $L_{27} = h^{-1}(L_1^R) \cap h^{-1}(L_5)$
   (i) $L_{28} = g(L_1^R)$
2. (4 ok) For each of the following languages give a DFA accepting it over the alphabet $\{0,1,2\}$ or $\{a,b,c\}$, depending on the alphabet of the language.

(a) L_{20}
(b) L_{21}
(c) L_{22}
(d) L_{23}
(e) L_{24}
(f) L_{25}
(g) L_{26}
(h) L_{28}

3. Give DFA's accepting the following languages over the alphabet $\Sigma = \{0, 1, 2, 3, 5, 6\}$:

   (a) the set of all strings beginning with a 1, 3 or 5, that, when the string is interpreted as an integer in base 9, is a multiple of 5 plus 2. For example:
   - strings 13,30,35,1333,1316,1613,513,563,55563 and 563635663 are in the language;
   - the strings 3,5,6, 135,136,20, 00, 022, 0020, 37 , 23, 5057, 223, 2325, 2375, 32222, 505, 22, 72, and 035 are not.

   (b) The set of all strings that ends with a 1, 3, or 5 and when the string is interpreted ***in reverse*** as an integer in base 9, is a multiple of 5 plus 2.
   - Examples of strings in the language are 31, 03, 53, 3331, 6131, 3161, 315, 365, 36555, and 36653636.
   - Examples of strings that are not in the language are: 3,5,6, 531,631,02, 00, 220, 0200, 73 , 32, 7505, 322, 5232, 5732, 22223, 505, 22, 27, and 530.

4. Consider the DFA with the following transition table:

		0	1
$\rightarrow$	$*$ 0	1	0
	1	2	1
	$*$ 2	3	2
	3	0	3

   Informally describe, as simple as possible, the language accepted by this DFA, and prove that your description is correct. You may use a proof based on induction on the length of an input string.

Marking Scheme: ***Marking Scheme***: The marks for assignments are added together.

- 0 marks: no submission or plagiarism, or almost no correct solution.
- 1 mark: most exercises are started and some are correct.

- 2 marks: most exercises are started and the majority are correct.
- 3 marks: almost all exercises are started and almost all are correct

Very Important: Verify your solutions using Grail; describe ***how do you think*** for each of the above exercises. Just giving the final solution without any explanation may result in a mark of 0 at the discretion of your instructor.

If you decide for a late submission, please, contact me, before the due date, because I will give the solutions to ***all*** exercises in class.