Assignment #2

Date Due: March 18, 2025

Total: 100 marks

We also have the following languages computed in Assignment #1: We have the following languages: $L_1 = \{$the set of all strings over the alphabet $\{0, 1, 2\}$ that begin with 1101$\}$,

$L_2 = \{$the set of all strings over the alphabet $\{0, 1, 2\}$ that end with 1011$\}$,

$L_3 = \{$the set of all strings over the alphabet $\{0, 1, 2\}$ with 101 being a subword$\}$,

$L_4 = \{$the set of all strings over the alphabet $\{0, 1, 2\}$ with an odd number of 2's$\}$,

$L_5 = \{$the set of all strings over the alphabet $\{0, 1, 2\}$ with an even number of 2's$\}$,

$L_6 = \{$the set of all strings over the alphabet $\{0, 1, 2\}$ having the forth symbol from the right end a 1$\}$,

$L_7 = \{$the set of all strings over the alphabet $\{0, 1, 2\}$ beginning with 1022$\}$,

$L_8 = \{$the set of all strings over the alphabet $\{0, 1, 2\}$ ending in 1022$\}$,

$L_9 = \{$the set of all strings over the alphabet $\{0, 1, 2\}$ with the number of 1's multiple of 6$\}$,

$L_{10} = \{$the set of all strings over the alphabet $\{0, 1, 2\}$ with the number of 1's multiple of 7$\}$,

$L_{11} = \{$the set of all strings over the alphabet $\{a, b\}$ with the number of a's multiple of 5$\}$,

$L_{12} = \{$the set of all strings over the alphabet $\{a, b\}$ with the number of b's multiple of 6$\}$,

$L_{13} = \{$the set of all strings over the alphabet$\{0, 1, 2\}$ consisting only of alternating groups of 21 and 11 (21 and 11 *alternates* at least once)$\}$,

and the following homomorphisms

$h : \{a, b\} \longrightarrow \{0, 1, 2\}^*$, $h(a) = 10$, $h(b) = 21$; $g : \{0, 1, 2\} \longrightarrow \{a, b\}^*$, $g(0) = aa$, $g(1) = b$, $g(2) = \varepsilon$; and $f : \{a, b, c\} \longrightarrow \{0, 1, 2\}^*$, $f(a) = 10$, $f(b) = 11$;

1. $L_{20} = L_1 \cap L_2$.
2. $L_{21} = 1011\Sigma^* \cap \Sigma^*1101$
3. $L_{22} = L_{13}$
4. $L_{23} = L_6$
5. $L_{24} = L_7 \cap L_8$
6. $L_{25} = L_{11} \setminus L_{12}$
7. $L_{26} = h^{-1}(L_4)$
8. $L_{27} = f^{-1}(L_1^R) \cap h^{-1}(L_5)$
9. $L_{28} = g(L_1^R)$

1. (60 marks) For each of the following languages give a regular expression generating them over the alphabet $\{0, 1, 2\}$ or $\{a, b, c\}$, depending on the description of the language (10 marks each):
   (a) L_{20}
   (b) L_{21}
   (c) L_{22}
   (d) L_{23}
   (e) L_{24}
   (f) L_{25}
   (g) L_{26}
   (h) L_{27}
   (i) L_{28}

2. (20 marks) Write regular expressions for the following languages over the alphabet $\Sigma = \{0, 1, 2, 3, 5\}$:
   (a) the set of all strings beginning with a **1, 2 or 3**, that, when the string is interpreted as an integer **in base 7, is a multiple of 6 plus 1**. For example:
      - strings 10,25,100,115,133,313,1015,1105,3013 and 200005 are in the language;
      - the strings 3,4, 21,02,04,23,50,113,135,3005,200001,3011,1103 and 035 are not.

   (b) The set of all strings that ends with an **1, 2, or 3** and when the string is interpreted ***in reverse*** as an integer **in base 7, is a multiple of 6 plus 1**.
      - Examples of strings in the language are 01,52,001,511,331,313,5101,5011,3103 and 500002
      - Examples of strings that are not in the language are: 3,4,12,20,40,32,05,311,531,5003,100002,1103,301 and 530.

3. (25 marks) Consider the DFA with the following transition table:

	0	1
$\rightarrow * 0$	1	3
1	2	1
$* 2$	1	2
$* 3$	3	4
4	4	3

   (a) (10 marks) Find the equivalent regular expression using the algorithms learned in class.

(b) (10 marks) Transform the regular expression into an ε-NFA

(c) (10 marks) Transform the ε-NFA into a DFA.

4. (25 marks) Check your results with Grail+ and comment on the Grail+ experiments (another 5 marks/test(language)).