Assignment #2

Date Due: November 3, 2025

Total: 100 marks

We also have the following languages computed in Assignment #1: We have the following languages:

$L_1 = \{$the set of all strings over the alphabet $\{0,1,2\}$ that begin with 0101$\}$,
$L_2 = \{$the set of all strings over the alphabet $\{0,1,2\}$ that end with 01011$\}$,
$L_3 = \{$the set of all strings over the alphabet $\{0,1,2\}$ with 110 being a subword$\}$,
$L_4 = \{$the set of all strings over the alphabet $\{0,1,2\}$ with an odd number of 0's$\}$,
$L_5 = \{$the set of all strings over the alphabet $\{0,1,2\}$ with an even number of 1's$\}$,
$L_6 = \{$the set of all strings over the alphabet $\{0,1,2\}$ having the fifth symbol from the right end a 0$\}$,
$L_7 = \{$the set of all strings over the alphabet $\{0,1,2\}$ beginning with 2102$\}$,
$L_8 = \{$the set of all strings over the alphabet $\{0,1,2\}$ ending in 2102$\}$,
$L_9 = \{$the set of all strings over the alphabet $\{0,1,2\}$ with the number of 1's multiple of 5$\}$,
$L_{10} = \{$the set of all strings over the alphabet $\{0,1,2\}$ with the number of 1's multiple of 7$\}$,
$L_{11} = \{$the set of all strings over the alphabet $\{a,b\}$ with the number of a's multiple of 6$\}$,
$L_{12} = \{$the set of all strings over the alphabet $\{a,b\}$ with the number of b's multiple of 5$\}$,
$L_{13} = \{$the set of all strings over the alphabet$\{0,1,2\}$ consisting only of alternating groups of 20 and 01 (20 and 01 *alternates* at least once)$\}$,

and the following homomorphisms

$h : \{a,b\} \longrightarrow \{0,1,2\}^*$, $h(a) = 10$, $h(b) = 21$; and $g : \{0,1,2\} \longrightarrow \{a,b\}^*$, $g(0) = a$, $g(1) = ba$, $g(2) = \varepsilon$.

1. $L_{20} = L_1 \cap L_2$.
2. $L_{21} = 01011\Sigma^* \cap \Sigma^*1010$
3. $L_{22} = L_{13}$
4. $L_{23} = L_6$
5. $L_{24} = L_7 \cap L_8$
6. $L_{25} = L_{11} \setminus L_{12}$
7. $L_{26} = h^{-1}(L_4)$
8. $L_{27} = h^{-1}(L_1^R) \cap h^{-1}(L_5)$
9. $L_{28} = g(L_1^R)$

1. (60 marks) For each of the following languages give a regular expression generating them over the alphabet $\{0,1,2\}$ or $\{a,b,c\}$, depending on the description of the language (10 marks each):

(a) L_{20}

(b) L_{21}

(c) L_{22}

(d) L_{23}

(e) L_{24}

(f) L_{25}

(g) L_{26}

(h) L_{27}

(i) L_{28}

2. (20 marks) Write regular expressions for the following languages over the alphabet $\Sigma = \{0, 1, 2, 4, 8\}$:

   (a) the set of all strings beginning with a **1, 2, or 4**, that, when the string is interpreted as an integer in base 9, is a multiple of 5 plus 2. For example:
   - strings 2,18,24,41,128,244,408,481,4881 and 24181 are in the language;
   - the strings 4, 8,02,04,42,44,124,404,48,2018,2418,882 and 035 are not.

   (b) The set of all strings that ends with an **1, 2, or 4** and when the ***reverse*** of the string is interpreted as an integer **in base 9, is a multiple of 5 plus 2**.
   - Examples of strings in the language are 2,81,42,14,821,442,804,184,1884 and 18142
   - Examples of strings that are not in the language are: 4,8,20,40,24,44,421,404,84,8102,8142,288 and 530.

3. (25 marks) Consider the DFA with the following transition table:

	0	1
$\rightarrow 0$	1	3
1	2	1
$*\ 2$	1	2
3	3	3

   (a) (10 marks) Find the equivalent regular expression using the algorithms learned in class.

   (b) (10 marks) Transform the regular expression into an ε-NFA

   (c) (10 marks) Transform the ε-NFA into a DFA.

4. (25 marks) Check your results with Grail+ and comment on the Grail+ experiments (another 5 marks/test(language)).