Assignment #1

Date Due: October 17, 2025

Total: 100 marks

We have the following languages:

$L_1 = \{$the set of all strings over the alphabet $\{0,1,2\}$ that begin with 0101$\}$,

$L_2 = \{$the set of all strings over the alphabet $\{0,1,2\}$ that end with 01011$\}$,

$L_3 = \{$the set of all strings over the alphabet $\{0,1,2\}$ with 110 being a subword$\}$,

$L_4 = \{$the set of all strings over the alphabet $\{0,1,2\}$ with an odd number of 0's$\}$,

$L_5 = \{$the set of all strings over the alphabet $\{0,1,2\}$ with an even number of 1's$\}$,

$L_6 = \{$the set of all strings over the alphabet $\{0,1,2\}$ having the fifth symbol from the right end a 0$\}$,

$L_7 = \{$the set of all strings over the alphabet $\{0,1,2\}$ beginning with 2102$\}$,

$L_8 = \{$the set of all strings over the alphabet $\{0,1,2\}$ ending in 2102$\}$,

$L_9 = \{$the set of all strings over the alphabet $\{0,1,2\}$ with the number of 1's multiple of 5$\}$,

$L_{10} = \{$the set of all strings over the alphabet $\{0,1,2\}$ with the number of 1's multiple of 7$\}$,

$L_{11} = \{$the set of all strings over the alphabet $\{a,b\}$ with the number of a's multiple of 6$\}$,

$L_{12} = \{$the set of all strings over the alphabet $\{a,b\}$ with the number of b's multiple of 5$\}$,

$L_{13} = \{$the set of all strings over the alphabet$\{0,1,2\}$ consisting only of alternating groups of 20 and 01 (20 and 01 *alternates* at least once)$\}$,

and the following homomorphisms

$h : \{a,b\} \longrightarrow \{0,1,2\}^*$, $h(a) = 10$, $h(b) = 21$; and $g : \{0,1,2\} \longrightarrow \{a,b\}^*$, $g(0) = a$, $g(1) = ba$, $g(2) = \varepsilon$.

1. (maximum 25 marks) Compute the languages (5 marks each)

   (a) $L_{20} = L_1 \cap L_2$.

   (b) $L_{21} = 01011\Sigma^* \cap \Sigma^*1010$

   (c) $L_{22} = L_{13}$

   (d) $L_{23} = L_6$

   (e) $L_{24} = L_7 \cap L_8$

   (f) $L_{25} = L_{11} \setminus L_{12}$

   (g) $L_{26} = h^{-1}(L_4)$

   (h) $L_{27} = h^{-1}(L_1^R) \cap h^{-1}(L_5)$

   (i) $L_{28} = g(L_1^R)$

2. (maximum 55 marks, 10 marks each) For each of the following languages give a DFA accepting it over the alphabet $\{0,1,2\}$ or $\{a,b,c\}$, depending on the alphabet of the language.

(a) L_{20}
(b) L_{21}
(c) L_{22}
(d) L_{23}
(e) L_{24}
(f) L_{25}
(g) L_{26}
(h) L_{28}

3. (20 marks) Give DFA's accepting the following languages over the alphabet $\Sigma = \{0, 1, 2, 4, 8\}$:

   (a) the set of all strings beginning with a 1, 2 or 4, that, when the string is interpreted as an integer in base 9, is a multiple of 5 plus 2. For example:
   - strings 2,18,24,41,128,244,408,481,4881 and 24181 are in the language;
   - the strings 4, 8,02,04,42,44,124,404,48,2018,2418,882 and 035 are not.

   (b) The set of all strings that ends with an **1, 2, or 4** and when the string is interpreted ***in reverse*** as an integer **in base 9, is a multiple of 5 plus 2**.
   - Examples of strings in the language are 2,81,42,14,821,442,804,184,1884 and 18142
   - Examples of strings that are not in the language are: 4,8,20,40,24,44,421,404,84,8102,8142,288 and 530.

4. (10 marks) Consider the DFA with the following transition table:

	0	1
$\rightarrow 0$	1	3
1	2	1
$*\ 2$	1	2
3	3	4
4	4	3

Informally describe, as simple as possible, the language accepted by this DFA, and prove that your description is correct. You may use a proof based on induction on the length of an input string.

The maximum is bounded to 115 marks.

Very Important: Verify your solutions using Grail (5 marks for each of exercises 2,3, and 4); describe ***how do you think*** for each of the above exercises. Just giving the final solution without any explanation may result in a mark of 0 at the discretion of your instructor.

If you decide for a late submission, please, contact me, before the due date, because I will give the solutions to ***all*** exercises in class.