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It is known that every random string in Kolmogorov-Chaitin sense (i.e.when the underlying complexity is measured by the blank-endmarker complexity) is Chaitin random (with respect to the self-delimiting complexity)—Chaitin's Theorem—but the converse implication is false—Solovay's Theorem. The "philosophy" of both approaches is similar: a random string is one whose complexity is approximately equal to its size. All known properties of Kolmogorov-Chaitin random strings are shared by Chaitin random strings and vice-versa. The only known semantic property distinguishing these definitions concerns the relation between random strings and random sequences: one can define random sequences in terms of Chaitin random strings, but not in terms of Kolmogorov-Chaitin random strings. The aim of this thesis is to compare these definitions, for both strings and numbers. Some constructive topologies will be used as the main instrument of investigation.

Here are some sample results:

- Given an arbitrary string x, is it possible to find a Chaitin non-random (random) string y having x as a prefix?

  The following two results can be used to answer the above questions:

  - The set of non random strings is dense in any topology generated by a recursive unbounded order.
  - The set of random strings is recursively rare in the topology generated by the prefix order, but dense in the topology generated by the infix order.

  Accordingly, given an arbitrary string x, it is always possible to find a Chaitin non-random string y having x as a prefix. The similar question, for a random extension, has, in general, a negative answer: there exist strings having no random extension. However, for every string x it is possible to find two strings u, v such that uxv is a Chaitin random string.

- The notion of Chaitin random number is introduced by means of positional representations in base p. The main result, which parallels the situation of random sequences, states the invariance under the change of (reasonable small) bases. Random naturals a p-normal, in any (reasonable small) base.

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